Contents, Sentences, and Possibilities

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Overview

The distinction between sentences and contents poses problems that withstand obvious solutions. These problems concern the relationship between contents and sentences, the ontological status that contents assume, and the relationship between contents and facts. The assumption that sentences create contents in the same way that, for example, the physical construction of a bridge creates the possibility of crossing a river solves these problems. In short, contents are possibilities; sentences do not name contents, values, or facts; and contents are as real as other physical conditions. A correspondence in the sense of the similarity between contents and facts is mediated by certain structures—either inner images and perceptions or data sets—that are comparable to each other and that represent contents and facts. The use of different notations for names of sentences and names of contents and reinterpreting Alfred Tarski's semantic theory of truth lead to a new program for the philosophy of logic. It is carried out in principle for propositional or content logic, class logic, and arithmetic. The distinction between rules of inference and theorems of logic is rendered redundant; higher level theorems replace rules of inference. Finally, this essay applies its new ideas to the logic of physics and provides new insights into the relationship between mathematical and physical theories. Although this essay does not confront methodological problems, its critical discussion is based on the methodological principle that empirically irrefutable philosophical theories are open to rational, critical discussion only in their logical relationship to problem situations.[1]

The Problem Situation

Common sense makes a distinction between sentences and contents. The following two fictitious statements regarding the same book illustrate this distinction.

> Karl: I found it difficult to follow the plot because my Spanish is not very good.
>
> Socrates: I found it difficult to follow the plot because there were so many characters involved in the plot.

Karl's insufficient knowledge of the language makes it difficult for him to grasp the content of the book. Socrates's difficulty involved, supposedly, the content itself.

The problems posed by this distinction require attention. What is the relationship between contents and sentences? Do sentences designate contents, or are sentences and contents identical? What is the ontological status of contents? Are contents figments of human imagination, or are they objective structures that exist independently of anyone's imagination? What is the relationship between contents and facts? Do sentences or contents correspond to facts, if they are true? Wherein lies this correspondence?

The identity thesis contains unacceptable consequences. Usually, double quotation marks are used to ascribe truth and falsity to sentences.

> "Socrates is the wisest man" is true.

The above sentence, as a whole, assigns truth to a sentence. How can a sentence be true? The question about the correspondence between sentences and facts, in the sense of their similarity, seems impossible to answer. Moreover, if the identity thesis were true, two different sentences could not have the same contents.

1. This methodology is formulated in Popper, "Status of Science and Metaphysics," 199.

There are unacceptable consequences of the identity thesis in another area as well. Theorems in propositional logic contain variables. The theorem—if (p and if p, then q), then q—contains the variables “p” and “q.” As a rule, constant names substitute variable names. In the mathematical theorem “a + b = b + a,” “a” and “b” are variable names of natural numbers. Substituting “4” for “a” and “5” for “b” leads to a true theorem of the calculus of natural numbers. Applied to the propositional theorem, however, this procedure does not lead to a true theorem. Substituting names of sentences, ““Socrates is the wisest man”” for “p” and ““Karl is ignorant”” for “q” leads to a result that is inadequate on formal grounds: If (“Socrates is the wisest man” and if “Socrates is the wisest man,” then “Karl is ignorant”), then “Karl is ignorant.” The mistake, one could assume, is to substitute names of sentences instead of sentences themselves. Substituting sentences seems straightforward but makes sentences names. As, for the identity thesis, sentences and contents are identical, sentences cannot name contents. What do sentences name? The identity thesis leaves this question unanswered.

Another possibility is to interpret “p” and “q” as variable names and sentences as constant names of contents. This assumption allows for the substitution of sentences for “p” and “q” and accepts contents as separate entities. Although this approach counters the above criticism, it leads to another insurmountable difficulty. Consider the following equivalence.[2]

P is true, if and only if p.

The letter “p” on the right-hand side of this equivalence is, according to Alfred Tarski, the metalanguage occurrence of an object language sentence, whose metalanguage name must be substituted for the variable “P.” If sentences were names of contents, the sentence itself would be the substitute for “P” to attribute truth or falsity to its content. Thus, the assumption that sentences are names of contents is incompatible with Alfred Tarski’s interpretation of the above equivalence.

Gottlob Frege explicitly rejects the idea that sentences designate contents, but upholds the argument that sentences designate things. He transfers this idea from words to sentences and assumes that every sentence designates a truth-value and is associated with a sense that he identifies as its content.[3] Tarski’s theory of truth and Frege’s approach are irreconcilable because one uses sentences as names, and the other, names of sentences instead. Furthermore, Frege denies the correspondence theory of truth and assumes that truth is a property that might be beyond definition and full understanding.[4]

Another tentative solution assumes that true sentences are names of facts or existing states of affairs.[5] Tarski’s semantic theory of truth focuses on sentences, and both approaches are reconcilable. The drawback is that it does not account for the relationship between contents and sentences. If sentences somehow express contents, this approach leaves them unnamed. Moreover, if sentences are names of facts, propositional theorems employing variables become theorems about facts, and propositional logic altogether vanishes.

2. Tarski, “The Semantic Conception of Truth,” 674.
3. Frege, “On Sense and Meaning,” 62-65.
4. Frege, *Logical Investigations*, 4, 6.
5. The position that true sentences name existing states of affairs is formulated in Tarski, “The Semantic Conception of Truth,” 667, but see footnote 6 there. Alfred Tarski does not identify facts and existing states of affairs, as is purported here.

These brief notes outline a problem situation that, allegedly, neither of the mentioned philosophical views solves. The solution to this problem situation surmounts these difficulties without causing new intractable problems and provides new tentative solutions to problems covered by its predecessors.[6]

The Tentative Solution

This section first outlines a general cosmological theory, applies it to a few seemingly different areas, and, finally, presents the solution to the problem situation posed in the previous section.

The cosmological theory, or "situational potentialism" for short, is a modification of Karl Popper's propensity interpretation of probability.[7] First, the modification is formulated and explained on the basis of an example following Popper's explanation of the propensity interpretation, and then, the additional idea necessary to preserve the full range of consequences of the propensity interpretation is introduced.[8] In short, situational potentialism is as follows.

The cosmos changes constantly. These changes are realizing possibilities embedded in generating (background) situations. Realizing possibilities are not the only inhabitants of the cosmos; there are mere possibilities that do not realize themselves. However, only realizing possibilities creates and destroys them. In this sense, the cosmos is inherently creative. Background situations are but bundles of realizing possibilities that progress only slowly in comparison to the foreground changes.

Tossing a coin on a wooden table with a flat surface serves as a first example. There are two possibilities: heads or tails. These two possibilities are not properties of the coin; they are created by the (background) situation. The background changes slowly in comparison to the tossing of the coin. Even the shape and condition of the coin are a part of the situation and contribute to the set of possibilities created by it. The wooden table is, in itself, a realizing possibility. Although it changes slowly in comparison to the tossing of the coin and in comparison to the coin itself, it dissipates eventually.

Occasionally, changes in the background situation result in an altered set of possibilities. Carving slots into the surface of the table creates a new possibility. The coin might slip into one of these slots. On the other hand, if the wooden table were replaced by a steel table, the set of possibilities would remain the same. Thus, not all aspects of the background situation are relevant for a particular set of possibilities.

Situational potentialism is independent of more substantial theories of the physical world and can thus be combined with other theories to draw a more complete picture of the universe. The argument that there is nothing else but the void and indestructible atoms is one such theory. Even this material cosmos possesses an infinite number of possibilities to arrange its objects. A table with an ink pot, a fountain pen and sheets of paper on it is a simple model of this cosmos. The writing utensils can be arranged in different positions on the table. The flat surface and the shape of the objects create distinct possibilities, so that even in a material world, there are immaterial things: possibilities.

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6. This methodological principle is formulated for empirically testable theories in Popper, "Conjectural Knowledge," 15, number (8).

7. Popper, *Realism and the Aim of Science*, section *53, and Popper, "A World of Propensities," *9–21*.

8. For these consequences, see Popper, *Quantum Theory and the Schism in Physics*, 125–130, "Particles, Waves and the Propensity Interpretation."

The main idea of the propensity interpretation of probability is the weight of possibilities to realize themselves, shown in stable relative frequencies of finite sequences upon repetition of the generating conditions. Propensities are properties of these conditions.[9] In its cosmological form, the propensity interpretation assumes that all possibilities have a tendency to realize themselves.[10]

Situational potentialism seems to be a weakened version of the propensity interpretation because it abstains from assigning weights to all possibilities and assumes that possibilities without a tendency to realize themselves exist. This seemingly slight modification widens the range of situations it covers. The ink pot and the fountain pen have zero propensity to move other places on the table. Nonetheless, there are many different possible positions for the items on the table. When a philosopher is around new possibilities and propensities are created, existing propensities change. The propensity of the pen to end up in another position within a reasonable period of time jumps to, for example, one. The philosopher would not have been able to move the pen without a preexisting possibility, but he did not create the possible positions for the pen. The propensity of the pen to move to another position and the propensity of the philosopher to move the pen involve two different possibilities. The first possibility does not have a tendency to realize itself—its propensity is zero; but the second possibility is endowed with a tendency to realize itself, and its propensity is not zero.

On the other hand, situational potentialism preserves the situational aspect of the propensity interpretation of probability. It assumes that possibilities are not properties of the ink pot or fountain pen, but are created by the (relevant) situation. This assumption amounts to a change from a cosmos of objects to a cosmos of interacting events embedded in background situations. To fully analyze the interaction of realizing possibilities, a theory of causality is necessary. This essay does not discuss theories of causality or the propensity interpretation any further, but confines the discussion to situational potentialism to formulate the solution for the problem situation described above.

A further example illustrates the point made above. Consider a tiled floor and five wooden boards. Since the boards are not joined in the shape of a container, nothing can be stored in them—at least, not on a macroscopic level. The presence of nails and a hammer create the possibility of building a box. When a skilled person realizes this possibility, the new and enduring possibility is formed to store things in the box. The boards do not rearrange themselves and certainly do not fix themselves with nails. Nonetheless, the mere possibility exists, even if nobody is around to realize it ever.

A blank sheet of paper, a crayon and an artist form the next example. Before the first stroke, the possibilities of drawings on that paper are unlimited. With each and every stroke, the drawing becomes increasingly detailed; each step reduces possibilities because after a while and several strokes, for instance, a rooster or tiger appears on the paper. Beginning, for example, with a long curved stroke eliminates the chance for a ladybug, but leaves the possibility for a snake or worm. Each and every stroke is the realization of a possibility inherently found in the situation, and each and every stroke eliminates and creates possibilities that change the situation. The result, the finished drawing, creates a new set of possibilities. It might evoke astonishment in its viewers, or wonder. It might be recognized as the drawing of a

9. Popper, *Realism and the Aim of Science*, 356.
10. Popper, “A World of Propensities,” 12.

tiger. The situations involving the wooden boards and the blank sheets of paper are very similar in light of situational potentialism, although the possibilities involved are quite different. The box is created by joining wooden boards. None of the boards offer the possibility of storage, but arranging them in a certain way creates a new possibility. The same holds true for the drawing. Every single stroke does not make the rooster or tiger. Several strokes arranged in a certain way, however, result in the drawing of an animal.

Situational potentialism in this rudimentary form, without a theory of causality, suffices to determine the relationship between sentences and contents. When sentences are concerned, naturally, words are involved. The roles of words and sentences are the same as the roles of the boards and the box and of the strokes and the drawing, respectively. Single words cannot provide the content, but arranged in a certain way, words form sentences that create new possibilities: contents. The main thesis is that sentences create possibilities and that contents are possibilities. More precisely, the whole relevant situation creates these possibilities and not just sentences by themselves. One important aspect of these situations is that words name objects in a constant way. Interpreting a word as the name of a different object changes the content of the sentence that includes the word. Whether sentences are spoken, signed or written is, on the other hand, not relevant to the content. All these presentations create the same content; but removing every other character from a printed version of a text probably destroys its content. As possibilities, contents are immaterial—in the sense explained above. Since they are bound to sentences, they are bound to sheets of paper; they are not part of anyone's mind; they exist independently and are objective in this sense. Moreover, sentences do not designate contents. Words designate objects outside language, but this does not mean that sentences designate things too. Words can designate inner pictures of a person's mind, but sentences built with these words create contents independent of minds.

At this point, the usefulness of the modification of the propensity interpretation of probability becomes clear. Situational potentialism assumes the existence of mere possibilities. Contents are not propensities but mere possibilities. Reading a book and picturing its content leave the content of the book untouched, just like crossing a river over a bridge leaves the possibility of crossing the river untouched. Nonetheless, the physical properties of a book play a role in the process of reading it. Analyzing this process requires a theory of causality and a full understanding of the mind-body problem. These issues are not discussed here.

Not all the possibilities attached to sentences are objective contents. Every spoken, signed or written utterance can evoke emotions. One consequent of the main thesis is that these emotions are not part of the objective content. These subjective aspects are real, but different from the content, and they must be described in psychological terms. The same holds true for the wooden box. Geometrical terms describe its shape; physical terms describe its stability. What kinds of terms are suitable to describe contents?

The final example leads the way to an answer. Consider the following two texts.

"Once upon a time, there was a young girl. Because she wore a hood,
everybody called her 'Little Riding Hood.'"

"Once upon a time, there was a young girl. Because she wore a red hood,
everybody called her 'Little Red Riding Hood.'"

The second text provides more details. In logical terms, the content of the second text excludes more. The content of the first version is compatible with mutually exclusive contents attributing any color to the hood. A subjective analysis misses this point because different readers create different pictures in their minds. Some readers may even be unaware of the difference between the two texts altogether. Therefore, logical terms must describe objective contents of sentences, and logic becomes the theory of contents, their logical forms, levels, and relations.

Ultimately, words do not create contents, nor do sentences designate things; but words designate things and sentences create contents. As sequences of characters, words are physical objects and therefore endowed with possibilities. The most important possibility is forming clusters that create new possibilities: contents. These contents are either true or false. Sentences do not have this property. Sentences are sequences of graphic symbols imprinted on sheets of paper. They provide the material substrate for contents. Logic is the theory of contents.

From now on, this essay uses single quotation marks to create names of contents and double quotation marks to create names of sentences. Although both types of quotation marks frame individual sentences, there is a significant difference between them. Double quotation marks create names of individual sentences or classes of similar sentences,[11] regardless of their contexts; single quotation marks create names of contents formed by the surrounding situation. This notation depends on the understanding that the relevant conditions outside the sentence do not change. Only in this case, the denoted content does not change. Some misunderstandings can be attributed to changes to relevant conditions. For example, when the assignment between expressions and denoted objects is changed, then singly quoted sentences denote different contents. This discussion will continue in later sections. A few examples shall introduce this notation.

"Socrates was the wisest man in Greece" is an English sentence. It is grammatically correct. "Sokrates war der weiseste Mann in Griechenland" is a German sentence. 'Socrates was the wisest man in Greece' is true. "Socrates was the wisest man in Greece" and "Sokrates war der weiseste Mann in Griechenland" create the same content. 'Socrates was the wisest man in Greece' and 'Sokrates war der weiseste Mann in Griechenland' are identical.

The remaining sections discuss this tentative solution. Leading questions are: Does the introduction of variable names of contents make variable use in logic and mathematic consistent with the rule that constant names substitute variable names? Can a philosophy of content logic adapt Tarski's semantic theory of truth without losing its insights into the hierarchical structure of language? These are not the only problems relevant to the critical discussion.

Critical Discussion

To fathom the problem-solving depth of the philosophical theory introduced above, this section confronts it with additional problems.

Does logic comprise axioms, theorems, and rules of inference?[12] Do arguments apply rules or theorems? If contents, rather than sentences, are true or false, can truth

11. The idea of classes of similar sentences or expressions is taken from Tarski, "The *S*emantic Conception of Truth," 666, footnote 5.

12. For an account of differences between rules and theorems, see Popper, "Calculi of Logic and Arithmetic," 203, and Tarski, *Introduction to Logic*, 47.

be explained as correspondence between contents and facts? Is there a difference between the use of mathematical and logical theorems? How do physical theories and arguments employ mathematical theorems?

The subsections below discuss problems of the correspondence theory of truth, propositional logic, class logic, and arithmetic. The final part uses ideas of Karl Popper's *The Logic of Scientific Discovery* to analyze the role mathematical and logical theorems play in physics.

Correspondence

The correspondence theory of truth explains truth as corresponding to facts. The philosophy of content logic focuses on correspondence in the sense of similarity and acknowledges that neither contents nor sentences correspond to facts in this sense directly, as they both are incomparable to facts. The following analysis assumes that correspondence and truth do not depend on subjective matters. Simply put, human beings do not make contents true; they make decisions on which assumptions to base their actions, but these decisions do not make the assumptions true. If this were the case, mistakes would be impossible. Truth is beyond human reach.

Where does the incomparability between contents, sentences and facts lie? And how can contents and sentences be made comparable? The incomparability lies in the fact that human beings—and this is valid for machines too—do not have direct access to contents and facts. A person who wants to assess the truth of a piece of content reads a sentence and pictures its content. This inner image is neither identical to the sentence nor the content nor the described fact. The person turns to the described fact, and his or her sense organs form sensory impressions of the fact. The inner image of the content and the sensory impressions of the fact are comparable. The correspondence between contents and facts is mediated by inner images and sensory impressions. Images and impressions act as proxies.

This theory of correspondence only works with contents and not with sentences. The inner image of a sentence is an image of letters and resembles the sensory impression of the sentence, as created by the sense organs. The inner image of a piece of content does not resemble the sensory impression of its sentence and is comparable to impressions formed by the sense organs attuned to the fact the content is supposed to describe.

In this sense, correspondence seems to be subjective. The next example shows that it is not subjective per se, but only if individuals refuse to submit their choices to objective tests.

Today's computers can visualize contents. A computer receives sentences in a computer-readable format as input and generates data output that represents the contents of the input. A photo camera produces data output that represents the part of reality the content is supposed to describe. Software compares the two datasets and calculates a degree of correspondence. Human intervention is not dispensable because an individual must program the computer.

This procedure explains correspondence between contents and facts as a relation between datasets. Human beings set objective standards for correspondence, but they do not fabricate it.

This section argued that the philosophy of content logic improves the problem situation of the correspondence theory of truth by explaining the correspondence between contents and facts in the sense of similarity, as mediated by correspondence between datasets or between inner images and sensory impressions representing contents and facts. Ultimately, statistical means define correspondence.

Content Logic

This section elaborates upon the idea that logic is the theory of contents, their logical forms, levels, and relations. It adapts Tarski's semantic theory of truth to the philosophy of content logic, introduces three basic and one derived logical form, defines the deducibility of contents, and analyzes the logical structure of arguments. Finally, it studies how theorems of content logic are proven.

The semantic theory of truth is, according to Alfred Tarski, formulated in a language that speaks about sentences and that to which they refer.[13] Any sentence can be part of a partial definition of truth.[14]

"Socrates is the wisest man" is true, if and only if Socrates is the wisest man.

The sentence on the left-hand side of this equivalence uses the name of a sentence to assign truth to it. The right-hand side consists of the metalanguage occurrence of the object language sentence whose name appears on the left-hand side.

The philosophy of content logic modifies this equivalence and opens the door to an entirely new approach to propositional logic and logic in general. The modified equivalence replaces the name of a sentence with the name of a piece of content.

'Socrates is the wisest man' is true, if and only if Socrates is the wisest man.

How does the philosophy of content logic interpret this modified equivalence? The name on the left-hand side designates a piece of object-level content, so the sentence on the left-hand side as a whole creates a piece of metalevel content. Therefore, the right-hand side of the sentence creates a piece of metalevel content too. Yet, there is an obvious difference. Whoever utters a sentence like "Socrates is the wisest man" claims that a piece of object-level content is true. The right-hand side of the modified equivalence makes this contention implicitly, while the left-hand side makes it explicitly. Because contents cannot state their own truth and, thus, claims to truth always refer to a lower level of content, all sentences and contents retain implicit claims on some level. The demand that all claims become explicit results in an infinite regression. The philosophy of content logic takes the modified equivalence as a way to switch between explicit and implicit representations of the same content. Applying the modified equivalence is nothing else but realizing a possibility inherent in language constructs and their environments.

The general form of the modified equivalence requires variable names of contents. The single quotation mark notation can be used for this purpose too, and the first general equivalence takes on the following form.

E1 'p' is true, if and only if p.

The interpretation of "'p'" is straightforward as a variable name of object-level contents; but "p" on the right-hand side seems to be an expression that must be substituted by a sentence. Since sentences are not names, "p" is not a sentence variable, and the status of "p" in E1 is unresolved. According to the above interpretation exemplified by individual sentences and contents, the right-hand sides of the equivalences create metalevel contents. This interpretation extends to "p." Therefore, "p" represents the metalevel content ''p' is true' implicitly. In short, "p" is an implicit and "'p'" an explicit variable name of object-level contents.

13. Tarski, "The Semantic Conception of Truth," sections 5, 8–10.
14. Ibid., 668.

The theory of logical forms and levels is at the core of the philosophy of content logic. The content ‘Socrates is the wisest man’ is in the logical form of a position. ‘Socrates is not ignorant’ is in the logical form of a negation. The position and the negation are basic logical forms of content logic. Other parts of logic—e.g., class logic—provide a fine-grained view of contents, but content logic cannot resolve the logical forms of the position and the negation any further. Logical forms in content logic are analogous to sentential functions in sentence logic.[15] Sentential functions contain so-called free variables. Sentences are derived from sentential functions either by binding free variables with quantifiers or by substituting constants for variables. Contents and forms, on the other hand, cannot be manipulated directly. To derive individual contents from logical forms, sentences must be modified. When variables substitute constants, the corresponding individual piece of content becomes a mere form. A piece of content is in the logical form F, if its sentence derives from F’s sentence by substitution. The semantic relationship of satisfaction prevails among forms and things, rather than sentential functions and things.[16] Three logical levels are important in the following analysis, and three different notations mark these levels.[17] Lowercase words “and,” “or,” “not,” “iff,” “if” characterize the object level; the symbols “∧,” “∨,” “→,” “↔” mark the metalevel; and uppercase words “AND,” “OR,” and “IF” represent the metametalevel.

The following two equivalences introduce the positive and the negative logical forms. “True” and “false” appear as undefined expressions.

E2 ‘p’ is true ↔ ‘not p’ is false.

E3 ‘p’ is false ↔ ‘not p’ is true.

The next primitive logical form is the conjunction.

E4 ‘p and q’ is true ↔ (‘p’ is true ∧ ‘q’ is true).

The content of the right-hand side—‘‘p’ is true ∧ ‘q’ is true’—is in the logical form of a conjunction on the metalevel. It is the explicit claim that two pieces of object-level content ‘p’ and ‘q’ are true. The claim to its own truth remains implicit. If it were made explicit, it would be a piece of metametalevel content. The content of the left-hand side—‘‘p and q’ is true’—is in the logical form of a position on the metalevel. It is the explicit claim that the piece of object-level content ‘p and q’ in the logical form of the conjunction is true. The claim to its own truth remains implicit too. Since E1–4 state the logical equivalence of metalevel contents, they are not definitions of logical symbols. The philosophy of content logic deals with contents first and only secondarily with expressions and symbols. More precisely, E4 defines the object level conjunction in metalevel terms. Formalizing a language, on the other hand, amounts to assigning logical forms to sentences with rules that employ names of sentences, expressions, and logical forms.[18] The syntactical form of a sentence in a formalized language indicates, therefore, the logical form of its content.

E5 and E6 derive the object level implication from previously introduced logical forms. E7 and E8 define it in metalevel terms.

15. For sentential functions see Tarski, *Introduction to Logic, 4.*

16. Naturally, the next step is to construct Tarski’s semantic definition of truth for contents instead of sentences. This is beyond the scope of this essay.

17. The method to mark levels by different notations is taken from Tarski, “The Concept of Truth,” 168–73. There, Tarski uses two different notations to mark the object level and the metalevel of sentences.

18. The conjunction could be formalized with this rule: The sequence of “p,” “and,” and “q” creates object-level content in the conjunctive logical form.

E5 ‘if p then q’ is true ↔ ‘not (p and not q)’ is true.
E6 ‘if p then q’ is true ↔ ‘p and not q’ is false.
E7 ‘if p then q’ is true ↔ (‘p’ is false ∨ ‘q’ is true).
E8 ‘if p then q’ is true ↔ (‘p’ is true → ‘q’ is true).

The above and similar equivalences define all the logical forms of content logic. Applying E1 and E8 to theorems of content logic poses a problem. A theorem known as “modus (ponendo) ponens” serves as an example.

if p and (if p then q), then q.

Substituting “Socrates is the wisest man” for “p” and “Karl is ignorant” for “q” leads to the following sentence.

if Socrates is the wisest man and (if Socrates is the wisest man, then Karl is ignorant), then Karl is ignorant.

Applying E1 renders the result explicit.

‘if Socrates is the wisest man and (if Socrates is the wisest man, then Karl is ignorant), then Karl is ignorant’ is true.

Applying E8 results in an implication that makes the problem acute.

‘Socrates is the wisest man and if Socrates is the wisest man, then Karl is ignorant’ is true → ‘Karl is ignorant’ is true.

The question is whether contents in the implicative logical form claim deducibility. The above metalevel content seems to claim that two individual pieces of content are deducible because its sentence uses the word “true.” Not all implicative contents express deducibility, though. Two independent axioms form a counterexample. ‘if (p and q) then (if p then q)’ is a theorem of content logic. The conjunction of two independent axioms is true, and because of the above theorem, their implication is true too. Thus, ‘if p then q’ cannot state deducibility. Only implications whose negations entail contradictions—logically true implications—state deducibility.[19] The following equivalence defines deducibility for object-level contents.

‘q’ is a logical consequence of ‘p’ ↔ ‘if p then q’ is logically true.

This equivalence explains why not all implications state deducibility. Content logic is too coarse to establish the logical truth of a piece of content in the implicative logical form ‘if p then q.’

The definition of deducibility interprets theorems of content logic as contents claiming deducibility without stating how arguments apply these theorems. The following analysis of example arguments based on content logic answers this question.

Karl is ignorant, because Socrates is the wisest man.

This argument contains two pieces of object-level content: ‘Socrates is the wisest man’ and ‘Karl is ignorant.’ Content logic cannot prove deducibility for these contents because its logical means are too weak. Rephrasing the argument changes the situation.

Socrates is the wisest man.
if Socrates is the wisest man, then Karl is ignorant.
Therefore, Karl is ignorant.

This form of the argument separates premise and conclusion and does not claim that a piece of content in the positional logical form follows from another position, but

19. An example proof is provided at the end of this section.

implicitly that the position ‘Karl is ignorant’ follows from the conjunction ‘Socrates is the wisest man, and if Socrates is the wisest man, then Karl is ignorant.’

The equivalence ‘‘p’ is true ↔ p’ transforms this argument into the explicit form.

‘Socrates is the wisest man’ is true.

‘if Socrates is the wisest man, then Karl is ignorant’ is true.

THEREFORE, ‘Karl is ignorant’ is true.

The use of “THEREFORE” indicates implicit claims that metalevel contents are true and deducible. A piece of metametalevel content presented in mixed notation states these claims explicitly.

IF ‘‘Socrates is the wisest man’ is true’ IS TRUE AND ‘‘if Socrates is the wisest man, then Karl is ignorant’ is true’ IS TRUE, THEN ‘‘Karl is ignorant’ is true’ IS TRUE.

Its logical form is: ‘IF ‘P’ IS TRUE AND ‘Q’ IS TRUE, THEN ‘R’ IS TRUE.’ Applying a metametalevel equivalence changes its logical form.

‘‘if p then q’ is true’ IS TRUE IFF IF ‘P’ IS TRUE, THEN ‘Q’ IS TRUE.

IF ‘‘Socrates is the wisest man’ is true’ IS TRUE AND (IF ‘‘Socrates is the wisest man’ is true’ IS TRUE, THEN ‘‘Karl is ignorant’ is true’ IS TRUE), THEN ‘‘Karl is ignorant’ is true’ IS TRUE.

This metametalevel content is now in the logical form of modus (ponendo) ponens.

IF ‘P’ IS TRUE AND (IF ‘P’ IS TRUE, THEN ‘Q’ IS TRUE), THEN ‘Q’ IS TRUE.

The substitution is “P” = “‘Socrates is the wisest man’ is true,” “Q” = “‘Karl is ignorant’ is true.”

Since modus ponens is true, the argument is valid. Moreover, the analysis—in terms of the philosophy of content logic—has replaced rules of inference with metametalevel theorems that are applied by substitution. One could assume that each argument has its own specific metametalevel content, stating deducibility of the conclusion from the premise. The next steps show that this is not necessarily the case. The use of propositional theorems as premises reduces the number of rules to one.[20] In content logic, it is the number of metametalevel contents that is reduced to one. First, applying the equivalences transforms the piece of metametalevel content into its metalevel form. The revised argument is still based on modus ponens. Second, performing the procedure with a different argument shows that just one piece of metametalevel content is necessary. The following alternating sequence of contents, equivalences and substitutions transforms the piece of metametalevel content in the logical form of modus ponens into a piece of metalevel content.

IF ‘‘Socrates is the wisest man’ is true’ IS TRUE AND (IF ‘‘Socrates is the wisest man’ is true’ IS TRUE, THEN ‘‘Karl is ignorant’ is true’ IS TRUE), THEN ‘‘Karl is ignorant’ is true’ IS TRUE.

‘P → Q’ IS TRUE IFF IF ‘P’ IS TRUE, THEN ‘Q’ IS TRUE.

“P” = “‘Socrates is the wisest man’ is true,” “Q” = “‘Karl is ignorant’ is true.”

IF ‘‘Socrates is the wisest man’ is true’ IS TRUE AND ‘‘Socrates is the wisest man’ is true → ‘Karl is ignorant’ is true’ IS TRUE, THEN ‘‘Karl is ignorant’ is true’ IS TRUE.

‘P ∧ Q’ IS TRUE IFF ‘P’ IS TRUE AND ‘Q’ IS TRUE.

“P” = “‘ Socrates is the wisest man’ is true,”

20. Popper, “Calculi of Logic and Arithmetic,” 203.

"Q" = "'Socrates is the wisest man' is true → 'Karl is ignorant' is true."
IF ''Socrates is the wisest man' is true ∧ ('Socrates is the wisest man' is true → 'Karl is ignorant' is true)' IS TRUE, THEN ''Karl is ignorant' is true' IS TRUE.
'P → Q' IS TRUE IFF IF 'P' IS TRUE, THEN 'Q' IS TRUE.
"P" = "'Socrates is the wisest man' is true ∧ ('Socrates is the wisest man' is true → 'Karl is ignorant') is true," "Q" = "'Karl is ignorant' is true."
''Socrates is the wisest man' is true ∧ ('Socrates is the wisest man' is true → 'Karl is ignorant' is true) → 'Karl is ignorant' is true' IS TRUE.
P IFF 'P' IS TRUE.
"P" = "'Socrates is the wisest man' is true ∧ ('Socrates is the wisest man' is true → 'Karl is ignorant' is true) → 'Karl is ignorant' is true."
'Socrates is the wisest man' is true ∧ ('Socrates is the wisest man' is true → 'Karl is ignorant' is true) → 'Karl is ignorant' is true.

A similar procedure transforms such metalevel content in the logical form of modus ponus into a metalevel position in either explicit or implicit form.

'if Socrates is the wisest man and (if Socrates is the wisest man, then Karl is ignorant), then Karl is ignorant' is true.
if Socrates is the wisest man and (if Socrates is the wisest man, then Karl is ignorant), then Karl is ignorant.

The revised version of the first argument has an additional premise in the implicative logical form that can be derived from the above metalevel position with E8.

'Socrates is the wisest man and if Socrates is the wisest man, then Karl is ignorant' is true.
'Socrates is the wisest man and (if Socrates is the wisest man, then Karl is ignorant)' is true → 'Karl is ignorant' is true.
THEREFORE, 'Karl is ignorant' is true.

A piece of metametalevel content in the logical form of modus ponens states the deducibility of the conclusion from the premise. This time, the substitution is "P" = "'Socrates is the wisest man and if Socrates is the wisest man, then Karl is ignorant' is true," "Q" = "'Karl is ignorant' is true."

IF ''Socrates is the wisest man and if Socrates is the wisest man, then Karl is ignorant' is true' IS TRUE AND (IF ''Socrates is the wisest man and if Socrates is the wisest man, then Karl is ignorant' is true' IS TRUE, THEN ''Karl is ignorant' is true' IS TRUE), THEN ''Karl is ignorant' is true' IS TRUE.

The first step seems futile; no obvious simplification has been realized. On the contrary, introducing another premise complicates the matter. Yet modus ponens on the metametalevel still states that the conclusion of the revised version of the argument follows from its premise. Applying the procedure to a second argument based on a different initial theorem shows that just one piece of metametalevel content is necessary.

'Socrates is the wisest man and Karl is ignorant' is true.
'Socrates is the wisest man and Karl is ignorant' is true → 'Karl is ignorant' is true.
THEREFORE, 'Karl is ignorant' is true.

The true implication stating deducibility of the conclusion from the premise resides, as shown above, on the metametalevel.

IF ‘‘Socrates is the wisest man and Karl is ignorant’ is true’ IS TRUE AND (IF ‘‘Socrates is the wisest man and Karl is ignorant’ is true’ IS TRUE, THEN ‘‘Karl is ignorant’ is true’ IS TRUE), THEN ‘‘Karl is ignorant’ is true’ IS TRUE.

This piece of metametalevel content is in the same logical form, modus ponens, as the analogous content of the first example. Without the additional premise, the metametalevel implication is in the logical form below.

IF ‘P’ IS TRUE AND ‘Q’ IS TRUE, THEN ‘Q’ IS TRUE.

The substitution is “P” = “‘Socrates is the wisest man’ is true,” “Q” = “‘Karl is ignorant’ is true.” Therefore, only one piece of metametalevel content, modus ponens, is necessary to apply theorems of propositional logic.

So far, the analysis has focused on individual arguments. Proofs of content logic theorems are nothing else but arguments. The example discussed here consists of two axioms and one theorem.[21]

A1 ‘if p then if q then p’ is true.

A2 ‘if (if p then (if p then q)) then (if p then q)’ is true.

T1 ‘if p then p’ is true.

The first substitution is “p” for “q” in A1 and A2.

A1' ‘if p then if p then p’ is true.

A2' ‘if (if p then (if p then p)) then p’ is true.

A piece of metametalevel content in the logical form of modus ponus states the deducibility of T1 from A1' and A2'. The substitution is “P” = “‘if p then if p then p’ is true,” “Q” = “‘if p then p’ is true.”

IF ‘‘if p then if p then p’ is true’ IS TRUE AND (IF ‘‘if p then if p then p’ is true’ IS TRUE, THEN ‘‘if p then p’ is true’ IS TRUE), THEN ‘‘if p then p’ is true’ IS TRUE.

If this claim to deducibility is made implicit, the proof of T1 assumes the form below.

A1' ‘if p then if p then p’ is true.

A2' ‘if p then if p then p’ is true → ‘if p then p’ is true.

T1' THEREFORE, ‘if p then p’ is true.

The above definition of deducibility uses the notion of logically true contents, defined as contents whose negations implied contradictions. All theorems of content logic are logically true in this sense. To demonstrate how these proofs appear in terms of content logic, first, theorem T1 is proven, and then modus ponens.

‘if p then p’ is false. (E3, E5)

‘p and not p’ is true.

THEREFORE, ‘if p then p’ is logically true.

‘if p and (if p then q), then q’ is false. (E3, E5)

‘p and (if p then q) and not q’ is true. (E4, E6)

‘p’ is true ∧ ‘p and not q’ is false ∧ ‘not q’ is true. (unspecified)

‘q’ is true ∧ ‘not q’ is true. (E4)

‘q and not q’ is true.

THEREFORE, ‘if p and (if p then q), then q’ is logically true.

21. Tarski, *Introduction to Logic*, 147–8.

In sum, the philosophy of content logic replaces rules of inference with metametalevel theorems. Simply put, there are no rules of inference. The use of metalevel equivalents reduces the number of necessary metametalevel contents to one. Analyzing arguments in terms of logical forms, levels, and substitutions suffices to describe both the application of theorems and their proofs. Metametalevel and metalevel equivalences are necessary to convert contents to different levels and forms. So far, the discussion of logical matters has remained within the range of content logic.

Class Logic

To extend the logical means to class logic, this section introduces one new logical form along with two relations between classes, studies the proof of a theorem, and, finally, examines an individual argument based on a theorem of class logic.

Content in the new subclass form states that the subclass relation holds between two classes. A sentence creating content in this logical form must contain names of classes and a symbol for the subclass relation. If “K,” “L,” and “M” are constant names of classes, and “⊂” indicates the subclass relation, the sentences “K ⊂ L,” “L ⊂ M,” and “K ⊂ M” can substitute “p,” “q,” and “r” in axioms and theorems of content logic. Although ‘if p and q, then r’ is not a theorem of content logic, the substitution “p” = “K ⊂ L,” “q” = “L ⊂ M,” and “r” = “K ⊂ M” leads to a true theorem of class logic.

A If K ⊂ L and L ⊂ M, then K ⊂ M.

The following proof is based on two axioms of class logic.[22] Both axioms and the theorem use an operation called “union,” which yields a class when applied to two classes. Its symbol is “∪,” so that “K ∪ L” names the union of the classes “K” and “L.” The conclusion of the proof is theorem T.

B ‘K ∪ L ⊂ M, iff K ⊂ M and L ⊂ M’ is true.

C ‘K ⊂ K’ is true.

T ‘K ∪ K ⊂ K’ is true.

The leading questions of the following analysis are: Which theorems of content logic are necessary to prove theorem T? How does the proof apply these theorems?

B ‘K ∪ L ⊂ M, iff K ⊂ M and L ⊂ M’ is true. (“L” = “K,” “M” = “K.”)

B' ‘K ∪ K ⊂ K, iff K ⊂ K and K ⊂ K’ is true.

B' is in the logical form of a position at the metalevel. The named object-level content is in the form of an equivalence. According to the equivalences introduced in the previous section, this metalevel position is equivalent to B''.

B'' ‘K ∪ K ⊂ K’ is true ↔ ‘K ⊂ K and K ⊂ K’ is true.

The left-hand side of B'' is identical to T. The next step is to prove the right-hand side of B'' with the help of C and a theorem of content logic.

TC ‘if p, then (p and p)’ is true. (“p” = “K ⊂ K.”)

TC' ‘if K ⊂ K, then (K ⊂ K and K ⊂ K)’ is true.

This position corresponds to the following implication.

TC'' ‘K ⊂ K’ is true → ‘K ⊂ K and K ⊂ K’ is true.

The antecedent of this implication is the second axiom C. Applying TC'' implicitly leads to the right-hand side of B''.

C ‘K ⊂ K’ is true.

THEREFORE, ‘K ⊂ K and K ⊂ K’ is true.

22. Tarski, *Introduction to Logic*, 141.

Applying TC" explicitly results in a different argument.

C ‘K ⊂ K’ is true.

TC" ‘K ⊂ K’ is true → ‘K ⊂ K and K ⊂ K’ is true.

THEREFORE, ‘K ⊂ K and K ⊂ K’ is true.

The piece of metametalevel content stating the deducibility is in the form of modus ponens.

IF ‘P’ IS TRUE AND (IF ‘P’ IS TRUE, THEN ‘Q’ IS TRUE), THEN ‘Q’ IS TRUE.

“P” = “‘K ⊂ K’ is true,” “Q” = “‘K ⊂ K and K ⊂ K’ is true.”

IF ‘‘K ⊂ K’ is true’ IS TRUE AND IF ‘‘K ⊂ K’ is true’ IS TRUE, THEN ‘‘K ⊂ K and K ⊂ K’ is true’ IS TRUE, THEN ‘‘K ⊂ K and K ⊂ K’ is true’ IS TRUE.

The last step of the proof is based on a different theorem of content logic.

‘if q and (p, iff q), then p’ is true.

“p” = “K ∪ K ⊂ K,” “q” = “K ⊂ K and K ⊂ K.”

‘if K ⊂ K and K ⊂ K and (K ∪ K ⊂ K, iff K ⊂ K and K ⊂ K), then K ∪ K ⊂ K’ is true.

‘K ⊂ K and K ⊂ K and (K ∪ K ⊂ K, iff K ⊂ K and K ⊂ K)’ is true → ‘K ∪ K ⊂ K’ is true.

The antecedent of this implication is the conjunction of the first conclusion—‘K ⊂ K and K ⊂ K’—with B'; the consequent is theorem T. The substitution “P” = “‘K ⊂ K and K ⊂ K and (K ∪ K ⊂ K, iff K ⊂ K and K ⊂ K)’ is true,” “Q” = “‘K ∪ K ⊂ K’ is true” in metametalevel modus ponens leads to a metametalevel content stating deducibility of T from suitable premises.

In sum, when proofs in class calculus use theorems of content logic explicitly, they use modus ponens in its metametalevel form implicitly. The remaining part of this section examines the relationship between theorems of class calculus and arguments.

All politicians are humans.

All humans are philosophers.

Thus, all politicians are philosophers.

This argument claims truth for its premises and deducibility of its conclusion from the premises implicitly. The general theorem involved is A.

A if K ⊂ L and L ⊂ M, then K ⊂ M.

Substituting constants for variables in A leads to the specific theorem A'.

A' if all politicians are humans and all humans are philosophers, then all politicians are philosophers.

Applying the equivalences previously introduced results in a piece of metalevel content claiming that a piece of object-level content in the positional logical form follows from a piece of object-level content in the conjunctive logical form.

A" ‘All politicians are humans and all humans are philosophers’ is true → ‘All politicians are philosophers’ is true.

A" and the explicit version of the example argument form a new argument.

‘All politicians are humans and all humans are philosophers’ is true.

‘All politicians are humans and all humans are philosophers’ is true → ‘All politicians are philosophers’ is true.

THEREFORE, ‘All politicians are philosophers’ is true.

The above premises and their conclusion are metalevel contents that state the truth and deducibility of object-level contents explicitly. The following metametalevel content in the logical form of modus ponens expresses the deducibility of the conclusion from the premises explicitly.

IF ‘P’ IS TRUE AND (IF ‘P’ IS TRUE, THEN ‘Q’ IS TRUE), THEN ‘Q’ IS TRUE.

“P” = “‘All politicians are humans and all humans are philosophers’ is true,”

“Q” = “‘All politicians are philosophers’ is true.”

IF ‘‘All politicians are humans and all humans are philosophers’ is true’ IS TRUE AND (IF ‘‘All politicians are humans and all humans are philosophers’ is true’ IS TRUE, THEN ‘‘All politicians are philosophers’ is true’ IS TRUE), THEN ‘‘All politicians are philosophers’ is true’ IS TRUE.

In summary, one piece of metametalevel content in the logical form of modus ponens suffices to analyze proofs and applications of theorems of class calculus.

Logic of Arithmetic

This section focuses on arithmetic theorems. First, it studies the relationship between an arithmetic theorem and an argument formulated in natural language. Second, it examines a proof of an arithmetic theorem in terms of content logic.

The new quantitative logical form simply gives the counted quantity of things. Usually, expressions like “the number of” or “seven” indicate this form in everyday language. The example consists of three pieces of quantitative content.

Socrates put one pearl of wisdom on the table.

Karl put three pearls of wisdom on the table.

Therefore, there are now four pearls of wisdom on the table.

There is a connection between this argument and the following axiom of arithmetic.

If x, y are numbers, there is a number z with x + y = z.

Since this theorem does not say anything about our argument, especially not that it is valid, the problem ahead involves examining this connection. More specifically, the problem is to reconstruct the logically true implicative content stating the deducibility of the conclusion from the premises. The simplest solution connects premises and the conclusion as antecedent and consequent, respectively.

If Socrates put one pearl of wisdom on the table and Karl put three pearls of wisdom on the table, then there are four pearls of wisdom on the table right now.

Is this piece of content logically true? Its logical form is ‘if p and q, then r.’ So, it is certainly not true from the perspective of content logic. Does it suffice to add the arithmetic theorem to the antecedent to make it true?

If Socrates put one pearl of wisdom on the table and Karl put three pearls of wisdom on the table, and for numbers x and y, there is the sum x + y, then there are four pearls of wisdom on the table right now.

Unfortunately, this is not enough. If pearls of wisdom, like drops of water, were to merge in close proximity, then the consequent would be false and the antecedent true. Therefore, the antecedent must state that pearls of wisdom behave in an additive way or, in other words, that arithmetic applies to them. The arithmetic theorem is not enough because it simply states that the number 1 + 3 = 4 exists, and not that the number of pearls on the table equals four. The logically true implication assumes, therefore, this form as follows.

If Socrates put one pearl of wisdom on the table and Karl put three pearls of wisdom on the table, and pearls of wisdom act like solid objects, so that their quantities add up, and for numbers x and y, there is the sum $x + y = z$, then there are four pearls of wisdom on the table right now.

This logically true implicative content can serve as one premise in an argument based on modus ponens. The argument's conclusion would be the consequent of the implication. The implication has three kinds of premises conjoined in the antecedent. First, there are contents describing the specific situation at hand—the initial conditions; second, contents characterize the part of the cosmos in a general way and are called theories; and third, there are mathematical theorems.[23] These contents are independent of each other. For example, consider that the true conclusion states there are eight pearls of wisdom on the table, and the initial conditions are true. An alternative theory could assume that pearls of wisdom somehow multiply in close proximity with a factor of two applied to the sum of pearls in the area. The last section resumes the discussion of these matters. The focus here is the use of arithmetic theorems.

To sum up, arithmetic theorems appear as premises in arguments consisting of contents in the quantitative logical form. A different question is whether this logical form provides the proper means to describe the cosmos at all. Progress in human science consists in part of new logical forms and theorems covering the argumentation with contents in these forms. Once a new logical form is available to scientists they can explore the new range of possible theories.

Yet another problem is determining the role theorems of content logic play in proofs of arithmetic theorems. The following proof[24] begins with a theorem of content logic called "reductio ad absurdum" and deploys the arithmetic theorem 'if $x < y$, then $y \nless x$.'

'if (if p, then not p), then not p' is true. (E8)
'if p, then not p' is true → 'not p' is true. ("p" = "$x < x$.")
'if $x < x$, then $x \nless x$' is true → '$x \nless x$' is true.
'if $x < y$, then $y \nless x$' is true. ("y" = "x.")
'if $x < x$, then $x \nless x$' is true.
THEREFORE, '$x \nless x$' is true.

A logically true piece of metametalevel content states the deducibility of the conclusion from the premises explicitly.

IF ''$x < x$' is true → '$x \nless x$' is true' IS TRUE AND (IF ''$x < x$' is true → '$x \nless x$' is true' IS TRUE, THEN ''$x \nless x$' is true' IS TRUE),
THEN ''$x \nless x$' is true' IS TRUE.

At first glance, this proof differs only slightly from a proof in syntactical terms.[25] The main difference is that metalevel theorems of content logic serve as premises in proofs of mathematical contents, and that one piece of metametalevel content, modus ponens, is necessary to make claims to deducibility explicit.

Logic of Physics

This section examines logical forms of and arguments with contents in physics. It develops a physical example theory and searches for logically true implications

23. This is an extension of Karl Popper's description of the situation. See Popper, *Logic of Scientific Discovery*, 38. The additional component is the mathematical content.
24. Tarski, *Introduction to Logic*, 157.
25. See Tarski, *Introduction to Logic*, 158, for a proof in syntactical terms.

covering arguments with these contents. It does not discuss methodological problems of empirical science, of which physics is just an example, but stays within the range of the empirico-deductive philosophy of science developed by Karl Popper in *The Logic of Scientific Discovery*. First, this section introduces two logical forms taken from this philosophy; second, it formulates background theories of space and time to elucidate these logical forms; and finally, it scrutinizes the relationship between mathematical and physical theories.

Although Popper did not intend an analysis of contents in the sense presented in this essay, the philosophy of content logic can absorb his results. Popper distinguishes the strictly universal from the singular existential and the purely existential logical forms.[26] Singular existential contents refer to a specific spatio-temporal region, for which they claim the occurrence of an event with certain properties. The strictly universal logical form, in contrast, refers to an unlimited spatio-temporal region and a potentially unlimited number of cases. Purely existential content claims the occurrence of an event in an unspecified spatio-temporal region.

Interpreted as strictly universal, the content ‘All ravens are white’ claims that ravens everywhere are always white. The content ‘Ravens living on planet earth right now are white’ is not strictly universal, for the set referred to with “Ravens living on our planet right now” has a finite number of elements located in a specific spatio-temporal region.

Singular existential contents do not follow from strictly universal ones. Nonetheless, they can contradict each other. The purely existential logical form serves as an intermediate form. From the singular existential content ‘There was a black raven on the spire of the Catholic church in Herongen on May 18, 2009’ follows ‘There are black ravens (somewhere sometime).’ The latter is a piece of content in the purely existential logical form, and it contradicts ‘There are no non-white ravens,’ which is equivalent to ‘All ravens are white.’

On the other hand, singular existential contents do not follow from purely existential ones, and purely existential contents do not follow from strictly universal ones. To sum up, purely existential contents are unilaterally verifiable by singular existential contents, and strictly universal contents are unilaterally falsifiable by singular existential contents.[27]

This is Popper’s description and analysis of logical forms in empirical science. In terms of the philosophy of content logic, the above logical forms are situated on the object level. Metalevel contents do not refer directly to space or time. They attribute truth or falsity to object-level contents. The position here is that these levels must not be confused. There are no singular existential metalevel contents.

Nonetheless, the theory of logical forms depends on a theory of space and time. Without at least a rudimentary theory of space and time, the singular existential and strictly universal logical forms are incomprehensible. The following philosophical theory provides the background to the theory of logical forms and to the analysis of the role of functions of real numbers and their theorems in physics.

> Time flows continuously. It is measurable with physical systems called “clocks.” The flow of time is the same for all physical systems. Temporal relations between events refer to this time and to a physically defined unit of time measurement. Space is coherent, and there is a physically defined unit for

26. For the following summary, see Popper, *Logic of Scientific Discovery*, sections 12, 13, 15 and 28.
27. Ibid., 49.

measurements of distances. Contents use positive real numbers and the metric d: $(x,y) \mapsto |x - y|$ to describe points in time, time spans, locations and spatial distances.

Scientists might picture time as a line, and the flow of time as a point moving along this line. The position here is that contents describing temporal or spatial relations are different from inner images, that time and space are not identical to geometrical objects, but that contents refer to geometrical objects to describe physical events. The content ‘This table is in the shape of a square’ refers to a geometrical shape to describe a specific spatio-temporal region.

These rudimentary theories of space and time are sufficient to analyze how contents describe the physical world. Contents refer to two positive real numbers. The first describes the point in time given in seconds, or fractions of seconds, as a period of time since a reference point set to zero; the second describes the spatial location given in meters, or fractions of meters, as a distance from a reference point set to zero. A simple physical example consists of an iron sphere falling down from the top of a wooden pole with a height of 45 meters. The top of the pole serves as a spatial reference point. A distance of 0 meters, therefore, indicates that the iron sphere is 45 meters above the ground. The point in time at which the iron sphere begins its free fall serves as a temporal reference point. A duration of 0 seconds, therefore, indicates that the iron sphere is at the beginning of its free fall. Since pairs of numbers are mathematical objects, their names do not create descriptive contents. The following sentence serves as a blueprint to generate descriptive contents for this physical system: The iron sphere is at the position y meters at the time x seconds.

Given two singular existential contents describing this iron sphere, the calculus of real numbers and the metric defined by d: $(x, y) \mapsto |x - y|$ determine the distance and duration of the fall. These calculations seem all too obvious. The justification lies in the background theories that assume space and time are described with positive real numbers. What is the logically true implicative content behind this argument?

If the iron sphere is at the position y_1 meters at the time x_1 seconds and at the position y_2 meters at the time x_2 seconds, then it fell $|y_1 - y_2|$ meters in $|x_1 - x_2|$ seconds.

Theorems of real numbers secure the existence of $|x_1 - x_2|$ and $|y_1 - y_2|$, but is this implicative content logically true? Strictly logically speaking, it is not true. A different metric could describe the physical space the iron sphere occupies throughout its free fall more accurately. To make the implicative content logically true, the assumption about the metric must become a part of the antecedent.

If the iron sphere is at the position y_1 meters at the time x_1 seconds and at the position y_2 meters at the time x_2 seconds, and if d: $(x, y) \mapsto |x - y|$ is the metric for space and time, then it fell the distance of $|y_1 - y_2|$ meters in the period of time of $|x_1 - x_2|$ seconds.

The mathematically defined metric does not provide any information about space or time. A background theory about space and time assumes that a specific metric of real numbers describes space and time correctly. Phrases like “at the position y in meters” and “at the point in time x in seconds” in the formulation of singular existential contents indicate that the corresponding contents provide an interpretation of numbers. The background theory is an independent assumption that is necessary to make the implication logically true. If measurements showed a different physical distance, the assumption that the metric d: (x, y) $\mapsto$ |x − y| described physical distances would be wrong.

In summary, to make the implicative content with the two singular existential contents logically true, the additional antecedent—that the metric d: (x, y) $\mapsto$ |x − y| gives distances in meters and time spans in seconds—is necessary. This assumption is part of the background theory.

Three theories serve as examples to study arguments with strictly universal contents.

T1 All mass objects in free fall to the surface of celestial bodies have at any time x in seconds the constant acceleration *a* in meters per squared second; *a* is specific to each celestial body.

T2 All mass objects in free fall to the surface of celestial bodies have at any time x in seconds the velocity $a\mathrm{x} + v$ in meters per second.

T3 All mass objects in free fall to the surface of celestial bodies have at any time x in seconds the spatial location $a\mathrm{x}^2 / 2 + v\mathrm{x} + s$ in meters.

T1–3 use functions, which as such do not describe the physical world. Mathematically speaking, they are but sets of, for example, pairs of real numbers. T1–3 and the background theory of space and time interpret these sets of numbers as time spans, acceleration, velocity, and spatial location. The background theory provides units of spatio-temporal distances defined in physical terms. Thus, T1–3 describe the physical world with mathematical means.

A logically true implicative content states the deducibility of T2 from T1 implicitly.

If all mass objects in free fall to the surface of celestial bodies have at any time x in seconds the constant acceleration *a* in meters per squared second, and if the function F: x $\mapsto a$ is the first derivation of the function G: x $\mapsto a\mathrm{x} + v$, then all mass objects in free fall to the surface of celestial bodies have at any time x in seconds the velocity $a\mathrm{x} + v$ in meters per second.

The second part of the conjunctive antecedent is a purely mathematical piece of content. It is a low level theorem of calculus. The function of mathematical theorems in arguments with physical theories is to ensure deducibility. Mathematical theorems are necessary premises in these arguments.

No singular existential content follows from physical theories alone.[28] To deduce them, singular existential contents that describe initial conditions are necessary as additional premises. T1 refers to celestial bodies. To test this theory, the following piece of content narrows it down to an observable spatial region.

The earth is a celestial body.

This additional assumption leads to the following conclusion.

All mass objects in free fall to the surface of the earth have at any time x in seconds the constant acceleration *a* in meters per squared second.

This theory is strictly universal in time but not in space, and it covers a potentially infinite set of objects. The logical theorem behind this argument belongs to class logic. The next premise narrows the theory down to one mass object.

This iron sphere has a mass of 1kg.

Three additional singular existential premises provide values for the constants *a*, *v* and *s* in T1–3.

28. See Popper, *Logic of Scientific Discovery*, 82, especially note *1.

The acceleration *a* of mass objects in free fall to the surface of the earth is 10 meters per squared second.

The initial velocity *v* of the iron sphere equals 0 meters per second.

The initial height *s* of its free fall is 0 meters.

The following content follows from the premises. It employs the function F: $x \mapsto x^2$ to describe the relation between the spatial and temporal locations of an object.

This iron sphere with the mass of 1 kg in free fall to the surface of the earth has at any time x in seconds the spatial location $5x^2$ meters.

Such content implies a singular existential piece of content that describes the free fall of the iron sphere.

This iron sphere with a mass of one kilogram takes three seconds to fall down from a height of 45 meters.

The last deduction requires mathematical theorems implicitly. Mere substitution leads to the expression "$5 \cdot 3^2$." The result, 45, follows from the mathematical theory.

To recapitulate, philosophical theories of space and time form the background of logical forms in physics. Mathematical theorems function as necessary premises in physics arguments. Finally, content in a strictly universal logical form provides a framework of slots that can be filled with any functional relation. The content in its logical form comes first and paves the way to a whole class of theories utilizing different functional relations.

Conclusion

The proposed philosophy of content logic provides a tentative solution to the hitherto unsolved problems posed at the beginning of this essay. Therefore, it succeeds in a field of study in which its competitors have failed. Identifying contents with immaterial possibilities created by sentences leads to the introduction of variable names of contents into logic and thus paves the way to consistent use of variables and constants in logic and mathematics. On this foundation, logic became the theory of contents and their logical forms, levels, and relations. Logical forms were analyzed in the contexts of propositional or sentential logic, in class logic, and arithmetic. The analyses of proofs and applications of theorems rendered rules of inference redundant, replacing them with logical theorems on higher content levels and thus providing a new tentative solution to problems covered by predecessors. Finally, the assumption that contents utilize functions to describe the physical world in ways determined by their logical forms shed light on the relationship between mathematics and physics.

Placed in a broader context, the philosophy of content logic points to a problem philosophy has yet to solve. The problem of the relationship between contents and sentences resembles the intricacies of the mind-body problem. Are minds identical to certain possibilities that materialize while the brain is active? Are minds immaterial? Do the mind and the brain interact? Since this essay does not provide a general theory of causality, understanding this relationship is at this point hopeless.

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